

The origin of the first and third generation fermion masses in a technicolor scenario

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Received: 24 July 2003 /

Published online: 26 November 2003 – © Springer-Verlag / Società Italiana di Fisica 2003

Abstract. We argue that the masses of the first and third fermionic generations, which are respectively of the order of a few MeV up to a hundred GeV, originate from a dynamical symmetry breaking mechanism leading to masses of the order $\alpha\mu$, where α is a small coupling constant, and μ , in the case of the first fermionic generation, is the scale of the dynamical quark mass (≈ 250 MeV). For the third fermion generation μ is the value of the dynamical techniquark mass (≈ 250 GeV). We discuss how this possibility can be implemented in a technicolor scenario, and how the mass of the intermediate generation is generated.

1 Introduction

The standard model is in excellent agreement with the experimental data. The only part of the model that is still obscure is the one responsible for the mass generation, i.e. the Higgs mechanism. In order to make the mass generation mechanism more natural there are several alternatives, where the most popular ones are supersymmetry and technicolor. In the first one the mass generation occurs through the existence of non-trivial vacuum expectation values of fundamental scalar bosons, while in the second case the bosons responsible for the breaking of gauge and chiral symmetry are composite.

Up to now the fermionic mass spectrum is the strongest hint that we have in order to unravel the symmetry breaking mechanism. A simple and interesting way to describe the fermionic mass spectrum is to suppose that the mechanism behind mass generation is able to produce a non-diagonal mass matrix with the Fritzsch texture [1]

$$M_f = \begin{pmatrix} 0 & A & 0 \\ A^* & 0 & B \\ 0 & B^* & C \end{pmatrix}. \quad (1)$$

This matrix is similar for the charged leptons, 1/3 and 2/3 charged quarks. The entry C is proportional to the mass of the third generation fermion, while the entry A is proportional to the mass of the lighter first generation. The diagonalization of such a mass matrix will determine the CKM mixing angles and the resulting diagonal mass matrix should reproduce the observed current fermion masses.

There are other possible patterns for the mass matrix and we choose the one of (1) just for simplicity. We call attention to the values of A and C . They must be of order of a few MeV and a hundred GeV, respectively. In models with a fundamental Higgs boson the values of A and C

are obtained due to adjusted vacuum expectation values (VEVs) or Yukawa couplings. In this way there is no natural explanation for the values of A and C ; they appear just as an ad hoc choice of couplings!

The question that we would like to discuss here is how we naturally can generate the scales A and C . In order to do so let us recall which are the mass scales in the standard model. In this model we have basically two natural mass scales: $\mu_{\text{QCD}} \approx 250$ MeV, which is the quantum chromodynamics (QCD) dynamical quark mass scale, and $v \approx 250$ GeV, the vacuum expectation value of the fundamental Higgs field responsible for the gauge symmetry breaking. As QCD is already an example of a theory with dynamical symmetry breaking we will also assume that technicolor (TC) models provide a more natural way to explain the gauge symmetry breaking [2, 3], i.e. at this level all the symmetry breaking mechanisms should be dynamical. Therefore we will not discuss a fundamental scalar field with VEV v but a composite scalar field characterized by $\mu_{\text{TC}} \approx 250$ GeV, which is the scale of the dynamical techniquark mass. Of course, at very high energies we possibly have other natural mass scales as the Planck one, the grand unified theory (GUT) scale M_{GUT} or the horizontal (family) symmetry mass scale M_h , although it is far from clear how such scales interfere with the values of A and C . Finally, in TC models we may also have the extended technicolor (ETC) mass scale M_{ETC} [4] upon which no constraint can be established above the 1 TeV scale [5]. In this work we will build a model where the scales A and C of (1) can be related respectively to the scales μ_{QCD} and μ_{TC} times some small coupling constant. The values of (1) will depend as little as possible on the very high energy mass scales like M_{GUT} , M_{ETC} , etc. The model will require a very peculiar dynamics for the TC theory as well as for QCD, and in this peculiarity in what concerns QCD the present approach differs from any other one that may

be found in the literature. In the next section we discuss which the dynamics is of non-Abelian theories that will lead to the desired relation between $A(C)$ and $\mu_{\text{QCD}}(\mu_{\text{TC}})$. In Sect. 3 we introduce a model assuming that its strongly interacting sector has the properties described in the previous section and show that the intermediate mass scale (B) of (1) appears naturally in such a scheme. In Sect. 4 we compute the fermion mass matrix. Section 5 contains some brief comments about the pseudo-Goldstone bosons that appear in our model, and we draw our conclusions in the last section.

2 The self-energy of quarks and techniquarks

In TC models the ordinary fermion mass is generated through the diagram shown in Fig. 1. In Fig. 1 the boson indicated by $SU(k)$ corresponds to the exchange of a non-Abelian boson, with coupling α_k to fermions (f) or technifermions (T). In the models found in the literature the role of the $SU(k)$ group is performed by the extended technicolor group and the boson mass is given by M_{ETC} . To perform the calculation of Fig. 1 we can use the following general expression for the techniquark (or quark) self-energy [6]:

$$\Sigma(p)_g = \mu \left(\frac{\mu^2}{p^2} \right)^\theta [1 + b g_{\text{TC(QCD)}}^2 (\mu^2) \ln(p^2/\mu^2)]^{-\gamma \cos(\theta\pi)} \quad (2)$$

where in the last equation we identified $\gamma = \gamma_{\text{TC(QCD)}}$ as the canonical anomalous dimension of the TC(QCD) mass operator, and μ is the dynamical fermion (TC or QCD) mass. The advantage of using such an expression is that it interpolates between the extreme possibilities for the technifermion (or quark) self-energy, i.e. when $\theta = 1$ we have the soft self-energy given by

$$\Sigma_s(p) = \frac{\mu^3}{p^2} [1 + b g_{\text{TC(QCD)}}^2 (\mu^2) \ln(p^2/\mu^2)]^\gamma, \quad (3)$$

which is the one obtained when the composite operator $\langle \bar{\psi}_i \psi_i \rangle \equiv \mu_i^3$ has canonical dimension and where i can indicate TC or QCD. When $\theta = 0$ operators of higher dimension may lead to the hard self-energy expression

$$\Sigma_h(p) = \mu [1 + b g_{\text{TC(QCD)}}^2 (\mu^2) \ln(p^2/\mu^2)]^{-\gamma}, \quad (4)$$

where γ must be larger than 1/2 and the self-energy behaves like a bare mass [7]. Therefore, no matter if the

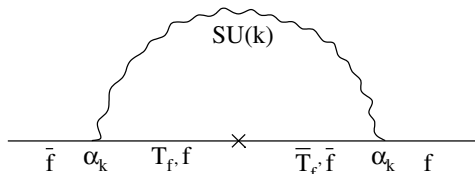


Fig. 1. Typical diagram contributing to the fermion masses of the first and third generation. The exchange of the boson indicated by $SU(k)$ plays the same role of an extended technicolor boson

dimensionality of the operators is responsible for the mass generation in technicolor theories, the self-energy can always be described by (2). In the above equations $g_{\text{TC(QCD)}}$ is the technicolor (QCD) coupling constant and $\gamma = \frac{3c_{\text{TC(QCD)}}}{16\pi^2 b}$, where

$$c_{\text{TC(QCD)}} = \frac{1}{2} [C_2(R_1) + C_2(R_2) - C_2(R_{\bar{\psi}\psi})],$$

with the quadratic Casimir operators $C_2(R_1)$ and $C_2(R_2)$ associated to the $R.H$ and $L.H$ fermionic representations of the technicolor (QCD) group, and $C_2(R_{\bar{\psi}\psi})$ is related to the condensate representation. b is the $g_{\text{TC(QCD)}}^3$ coefficient of the technicolor (QCD) group β function. The complete equation for the dynamical fermion mass displayed in Fig. 1 is

$$sm_f = a_k \int dq^4 \left(\frac{\mu^2}{q^2} \right)^\theta \times \frac{g_k^2(q) \left[1 + b_{\text{TC(QCD)}} g_{\text{TC(QCD)}}^2 \ln \left(\frac{q^2}{\mu^2} \right) \right]^{-\delta}}{(q^2 + M_k^2)(q^2 + \mu_{\text{TC(QCD)}}^2)}, \quad (5)$$

where we define $a_k = \frac{3C_{2k}\mu_{\text{TC(QCD)}}}{16\pi^4}$.

In the last equation C_{2k} is the Casimir operator related to the fermionic representations of the $SU(k)$ (or ETC) group connecting the different fermions (TC or QCD), g_k and M_k are the respective coupling constant and gauge boson mass, a factor $\mu_{\text{TC(QCD)}}$ remained in the fermion propagator as a natural infrared regulator and $\delta = \gamma \cos \theta\pi$, $g_k^2(q)$ is assumed to be given by

$$g_k^2(q^2) \simeq \frac{g_k^2(M_k^2)}{\left(1 + b_k g_k^2(M_k^2) \ln \left(\frac{q^2}{M_k^2} \right) \right)}. \quad (6)$$

Note that in (5) we have two terms of the form $[1 + b_i g_i^2 \ln q^2]$ where the index i can be related to TC(QCD) or $SU(k)$. To obtain an analytical formula for the fermion mass we will consider the substitution $q^2 \rightarrow \frac{x M_k^2}{\mu_i^2}$, and we will assume that $b_k g_k^2(M_k) \approx b_{\text{TC(QCD)}} g_{\text{TC(QCD)}}^2(M_k)$, which will simplify considerably the calculation. Knowing that the $SU(k)$ group usually is larger than the TC(QCD) one, we numerically computed the error in this approximation for a few examples found in the literature. The resulting expression for m_f will be overestimated by a factor 1.1–1.3 and is given by

$$m_f \simeq \frac{3C_{2k} g_k^2(M_k) \mu}{16\pi^2} \left(\frac{\mu^2}{M_k^2} \right)^\theta \times \left[1 + b_{\text{TC(QCD)}} g_{\text{TC(QCD)}}^2 \ln \frac{M_k^2}{\mu^2} \right]^{-\delta} I, \quad (7)$$

where

$$I = \frac{1}{\Gamma(\sigma)} \int_0^\infty d\sigma \sigma^{\epsilon-1} e^{-\sigma} \frac{1}{\theta + \rho\sigma},$$

with $\rho = b_{\text{TC(QCD)}} g_{\text{TC(QCD)}}^2 (M_k)$ and $\epsilon = \delta + 1 = \gamma \cos \theta \pi + 1$. To obtain (7) we made use of the following Mellin transform:

$$\left[1 + \kappa \ln \frac{x}{\mu^2} \right]^{-\epsilon} = \frac{1}{\Gamma(\epsilon)} \int_0^\infty d\sigma e^{-\sigma} \left(\frac{x}{\mu^2} \right)^{-\sigma \kappa} \sigma^{\epsilon-1}. \quad (8)$$

Finally, we obtain

$$m_f \simeq \frac{3C_{2k} g_k^2 (M_{\text{ETC}}) \mu}{16\pi^2} \left(\frac{\mu^2}{M_{\text{ETC}}^2} \right)^\theta F(\cos \theta \pi, \gamma, \rho), \quad (9)$$

where

$$\begin{aligned} & F(\cos \theta \pi, \gamma, \rho) \\ &= \Gamma \left(-\gamma \cos(\theta \pi), \frac{\theta}{\rho} \right) \exp \left(\frac{\theta}{\rho} \right) \frac{1}{\rho} \left(\frac{\theta}{\rho} \right)^{\gamma \cos(\theta \pi)} \\ & \times \left[1 + b_{\text{TC(QCD)}} g_{\text{TC(QCD)}}^2 \ln \frac{M_k}{\mu^2} \right]^{-\gamma \cos(\theta \pi)}. \end{aligned}$$

Simple inspection of the above equations shows that $\theta = 0$ leads us to the relation that we are looking for i.e.

$$C \propto g_k^2 \mu_{\text{TC}}, \quad (10)$$

which gives masses of $O(\text{GeV})$. If the $SU(k)$ (or ETC) bosons connect quarks to other ordinary fermions we also have

$$A \propto g_k^2 \mu_{\text{QCD}}, \quad (11)$$

which are masses of a few MeV. To obtain (10) and (11) we neglected the logarithmic term that appears in (9). In principle there is no problem to assume the existence of a TC dynamical self-energy with $\theta = 0$. There are TC models where it has been assumed that the self-energy is dominated by higher order interactions that are relevant at or above the TC scale, leading naturally to a very hard dynamics [8,9]. The existence of a hard self-energy in QCD is the unusual ingredient that we are introducing here. Usually it is assumed that such a solution is not allowed due to a standard operator product expansion (OPE) argument [10]. This argument does not hold if there are higher order interactions in the theory or a non-trivial fixed point of the QCD (or TC) β function [11]. There are many pros and cons in this problem which we will not repeat here [12], but we just argue that several recent calculations of the infrared QCD (or any non-Abelian theory) are showing the existence of an IR fixed point [13] and the existence of a gluon (or technigluon) mass scale which naturally leads to an IR fixed point [14]. The existence of such a mass scale seems to modify the structure of chiral symmetry breaking [15]. This fact is not the only one that may lead to a failure of the standard OPE argument. For instance, the effect of dimension two gluon condensates, if they exist [16], can also modify the dynamics of chiral symmetry breaking and this possibility has not been investigated up to now. Therefore it seems that we still do not have a full understanding of the IR behavior of the non-Abelian theories, which can modify the behavior of the self-energies that we are dealing with.

According to this we will just assume that such behavior can occur in TC as well as in QCD. In how far this is a bad or good assumption will certainly be reflected in the fermionic spectrum that we shall obtain. Finally, this is our only working hypothesis and will lead us to the following problem: How can we prevent the coupling of the first and second fermionic generations to the technifermions? A model along this line is proposed in the next section.

3 The model

3.1 The fermionic content and couplings

According to the dynamics that we proposed in the previous section, which consists in a self-energy with $\theta = 0$ in (2), and as the different fermion masses will be generated due to the interaction with different strong forces, we must introduce a horizontal (or family) symmetry to prevent the first and second generation ordinary fermions to couple to technifermions at leading order. The lighter generations will couple only to the QCD condensate or only at higher loop order in the case of the TC condensate. Using the hard expression for the self-energy (4) the fermion masses will depend only logarithmically on the masses of the gauge bosons connecting ordinary fermions to technifermions. Therefore we may choose a scale for these interactions of the order of a GUT scale, without the introduction of large changes in the value of the fermion masses. We stress again that the only hypothesis introduced up to now is the dynamics described in the previous section. On the other hand, as we shall see in the sequence, we will substitute the needed extended technicolor group by the existence of a quite expected unified theory containing TC and the standard model (SM) at a GUT scale.

There is also another advantage in our scheme. It will be quite independent of the physics at this ‘‘unification’’ scale and will require only a symmetry (horizontal) preventing the leading order coupling of the light fermion generations to technifermions. Finally, the horizontal symmetry will be a local one, although we expect that a global symmetry will also lead to the same results. We consider a unified theory based on the $SU(9)$ gauge group, containing a $SU(4)_{\text{TC}}$ TC group (stronger than QCD) and the standard model, with the following anomaly free fermionic representations [17]:

$$5 \otimes [9, 8] \oplus 1 \otimes [9, 2], \quad (12)$$

where the [8] and [2] are antisymmetric under $SU(9)$. Therefore the fermionic content of these representations can be decomposed according to the group product $SU(4)_{\text{TC}} \otimes SU(5)_{\text{GG}}$ ($SU(5)_{\text{GG}}$ is the standard Georgi–Glashow GUT [18]) as follows:

[9, 2]

$$(1, 10) = \begin{pmatrix} 0 & \bar{u}_{iB} & -\bar{u}_{iY} - u_{iR} - d_{iR} \\ -\bar{u}_{iB} & 0 & \bar{u}_{iR} - u_{iY} - d_{iY} \\ \bar{u}_{iY} & -\bar{u}_{iR} & 0 & -u_{iB} - d_{iB} \\ u_{iR} & u_{iY} & u_{iB} & 0 & \bar{e}_i \\ d_{iR} & d_{iY} & d_{iB} & -\bar{e}_i & 0 \end{pmatrix},$$

$$(4, 5) = \begin{pmatrix} Q_{iR} \\ Q_{iY} \\ Q_{iB} \\ \bar{L}_i \\ \bar{N}_i \end{pmatrix}_{\text{TC}}, \quad (\bar{6}, 1) = N_i,$$

[9, 8]

$$(1, \bar{5}) = \begin{pmatrix} \bar{d}_{iR} \\ \bar{d}_{iY} \\ \bar{d}_{iB} \\ e_i \\ \nu_{e_i} \end{pmatrix}, \quad (1, \bar{5}) = \begin{pmatrix} \bar{X}_{Rk} \\ \bar{X}_{Yk} \\ \bar{X}_{Bk} \\ N_{E_k} \\ N_{E_k} \end{pmatrix}_i,$$

$$(\bar{4}, 1) = \bar{Q}_{i\varepsilon}, L_i, N_{iL},$$

where $\varepsilon = 1, \dots, 3$ is a color index and $k = 1, \dots, 4$ indicates the generation number of exotic fermions that must be introduced in order to render the model anomaly free. These fermions will acquire masses of the order of the grand unified scale. We are also indicating a generation (or horizontal) index $i = 1, \dots, 3$, that will appear due to the necessary replication of families associated to a $SU(3)_H$ horizontal group.

This model is a variation of a model proposed by one of us many years ago [19]. The mass matrix of (1) will be formed according to the representations of the strongly interacting fermions of the theory under the $SU(3)_H$ group. The technifermions form a quartet under $SU(4)_{\text{TC}}$ and the quarks are triplets of QCD. The technicolor and color condensates will be formed at the scales μ_{TC} and μ_{QCD} in the most attractive channel (MAC) [20] of the products $\bar{\mathbf{4}} \otimes \mathbf{4}$ and $\bar{\mathbf{3}} \otimes \mathbf{3}$ of each strongly interacting theory. We assign the horizontal quantum numbers to technifermions and quarks such that these same products can be decomposed in the following representations of $SU(3)_H$: $\bar{\mathbf{6}}$ in the case of the technicolor condensate, and $\mathbf{3}$ in the case of the QCD condensate. For this it is enough that the standard left-handed (right-handed) fermions transform as triplets (antitriplets) under $SU(3)_H$, assuming that the TC and QCD condensates are formed in the $\bar{\mathbf{6}}$ and in the $\mathbf{3}$ of the $SU(3)_H$ group. This is consistent with the MAC hypothesis [20] although a complete analysis of this problem is out of the scope of this work.

The above choice for the condensation channels is crucial for our model, because the TC condensate in the representation $\bar{\mathbf{6}}$ (of $SU(3)_H$) will interact only with the third fermionic generation while the $\mathbf{3}$ (the QCD condensate) will interact only with the first generation. In this way we can generate the coefficients C and A , respectively, of (1), because when we add these condensates (VEVs) and write them as a 3×3 matrix we will end up (*at leading order*) with

$$M_f = \begin{pmatrix} 0 & A & 0 \\ A^* & 0 & 0 \\ 0 & 0 & C \end{pmatrix}. \quad (13)$$

This problem is very similar to the one proposed by Berezhiani and Gelmini et al. [21] where the VEVs of the fundamental scalars are substituted by condensates. The new

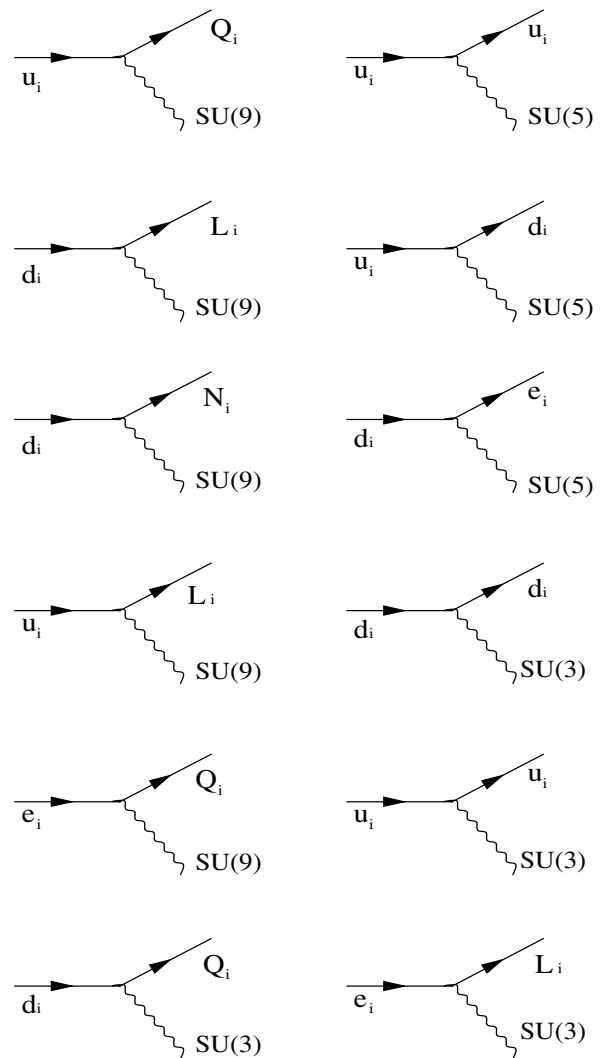


Fig. 2. Couplings of ordinary fermions and technifermions to the gauge bosons of $SU(9)$, $SU(5)_{GG}$ and $SU(3)_H$ which are relevant for the generation of fermion masses

couplings generated by the unified $SU(9)$ group and by the horizontal symmetry $SU(3)_H$ are shown in Fig. 2. With the couplings shown in Fig. 2 we can determine the diagrams that are going to contribute to the 2/3 and 1/3 charged quark masses as well as to the charged lepton masses. These diagrams are respectively shown in Fig. 3 to 5. It is important to observe the following in the above figures: The second generation fermions obtain masses only at the two loop order. This mass will be proportional to μ_{TC} times two small couplings (g_h and g_9 ; respectively the $SU(3)_H$ and $SU(9)$ coupling constants). It will also be non-diagonal in the $SU(3)_H$ indices. The first generation fermions obtain masses only due to the QCD condensate whereas the third generation ones couple directly to the TC condensates. Due to the particular choice of representations under the unified theory containing TC and the standard model we end up with more than one mass diagram for several fermions. This is particularly interesting: the way the fermions of the first generation obtain masses. In some of the diagrams of

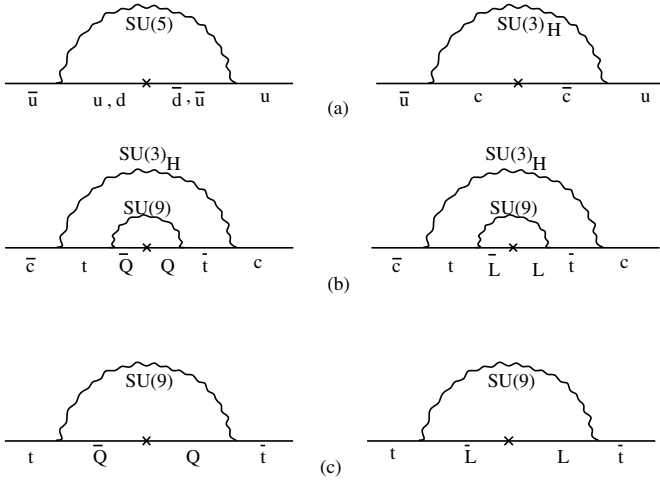


Fig. 3. Diagrams contributing to the charge 2/3 quark masses. In (a) we indicate by $SU(5)$ the exchange of a boson that belongs to the $SU(9)$ group, but that would also appear in the minimal $SU(5)$ GUT

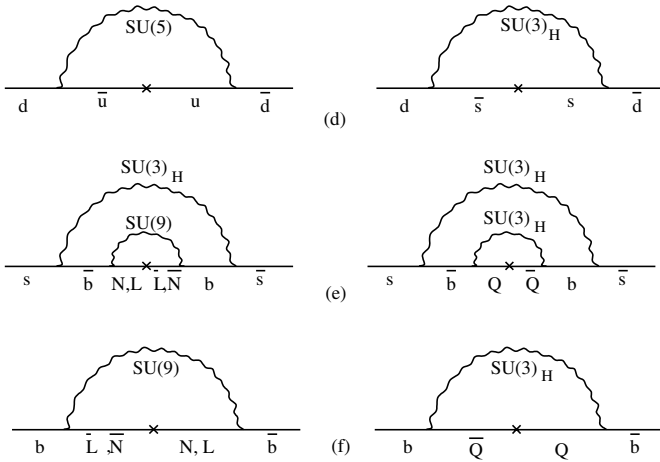


Fig. 4. Diagrams contributing to the mass generation of 1/3 charged quarks

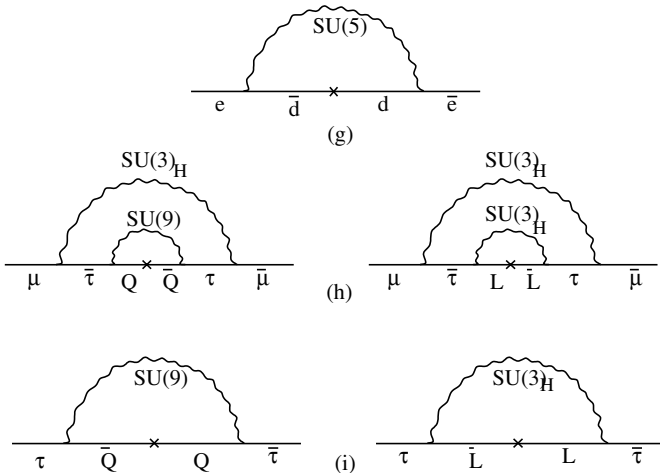


Fig. 5. Diagrams contributing to lepton masses

the above figures we show a boson that is indicated by $SU(5)$. This boson belongs to the $SU(9)$ group but would also appear in the standard $SU(5)_{\text{GG}}$ GUT. For example, the electron only couples to the d quark (and to the QCD condensate) through a $SU(5)_{\text{GG}}$ gauge boson existent in the Georgi–Glashow minimal GUT, whereas the u and d can connect to the second generation through the horizontal symmetry gauge bosons. We also expect other diagrams at higher order in g_h and/or g_9 that are not drawn in these figures.

3.2 The composite Higgs system

We can also observe that the second generation fermions will be massive, not by looking at the diagrams of Fig. 3 to 5, but studying the composite Higgs system. By this we mean that the QCD and TC condensates act as if we had two composite bosons represented by the fields η and φ . In principle this system could be described by the following effective potential:

$$V(\eta, \varphi) = \mu_\eta^2 \eta^\dagger \eta + \lambda_\eta (\eta^\dagger \eta)^2 + \mu_\varphi^2 \varphi^\dagger \varphi + \lambda_\varphi (\varphi^\dagger \varphi)^2, \quad (14)$$

in such a way that we can identify the VEVs (given by the ratio of masses and couplings):

$$v_\eta^2 = -\frac{\mu_\eta^2}{\lambda_\eta}, \quad v_\varphi^2 = -\frac{\mu_\varphi^2}{\lambda_\varphi}, \quad (15)$$

to the QCD and TC vacuum condensates. The bosons represented by η and φ , respectively, are related to the system of composite Higgs bosons formed in the representations $\mathbf{3}$ and $\bar{\mathbf{6}}$ of the horizontal group. Such a supposition is quite plausible if we consider the results of [8, 9], where it was shown that the interactions of a composite Higgs boson is very similar to the ones of a fundamental boson. Our intention is to show that such a system leads to an intermediate mass scale and to a mass matrix identical to (1).

The VEVs of QCD and technicolor, due to the horizontal symmetry, can be written respectively in the following form [21]:

$$\langle \eta \rangle \sim \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad \langle \varphi \rangle \sim \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & v_\varphi \end{pmatrix}, \quad (16)$$

and will be of the order of 250 MeV and 250 GeV. It is instructive at this point to observe what fermionic mass matrix we can obtain with the VEVs of (16). We can assume that the composite scalars η and φ have ordinary Yukawa couplings [1, 21] to the fermions described by the following effective Yukawa lagrangian:

$$\mathcal{L}_Y = a \bar{\Psi}_{L\lambda}^i \eta_\lambda^k U_R^j \epsilon_{ijk} + b \bar{\Psi}_{L\lambda}^i \varphi^{ij} U_R^j, \quad (17)$$

where Ψ and U are the ordinary fermion fields. λ is a weak hypercharge ($SU(2)_w$) index, for instance, $\lambda = 1$ represents charge 2/3 quarks and $\lambda = 2$ corresponds to the charge 1/3 quarks; i, j and k indicate the components of the composite

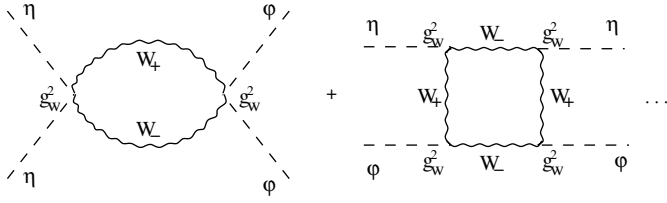


Fig. 6. Higher order corrections coupling the η and φ composite bosons

scalar bosons of the representations $\mathbf{3}$ and $\bar{\mathbf{6}}$ of $SU(3)_H$, and a and b are the coupling constants. Substituting the VEVs of (16) in the Yukawa lagrangian for the charge 2/3 quarks, we obtain

$$\mathcal{L}_Y = a\bar{c}_L v_\eta u_R - a\bar{u}_L v_\eta c_R + b\bar{t}_L v_\varphi t_R, \quad (18)$$

leading to a mass matrix in the (u, c, t) basis which is given by

$$\bar{m}^{\frac{2}{3}} = \begin{pmatrix} 0 & -av_\eta & 0 \\ av_\eta & 0 & 0 \\ 0 & 0 & bv_\varphi \end{pmatrix}. \quad (19)$$

The main point of the model is that the fermions of the third generation obtain large masses because they couple directly to the technifermions, while the ones of the first generation obtain masses originating from the ordinary condensation of QCD quarks. Having this picture in mind we can now see that the most general VEV for this system includes the mass generation for the intermediate family.

It is important to verify that there is no way to prevent the coupling at higher order of the different composite scalar bosons with $SU(3)_H$ quantum numbers. Examples of such couplings are shown in Fig. 6.

The diagrams of Fig. 6 will produce new terms for the effective potential of our composite system; therefore we must add to (14) the following terms:

$$V_2(\eta, \varphi) = \Pi\eta^\dagger\eta\varphi^\dagger\varphi + \delta\eta^\dagger\varphi\eta\varphi^\dagger + \dots \quad (20)$$

The introduction of this expression in the potential of (14) will shift the VEVs generated by the effective fields η and φ , and the VEV associated to the field η will be shifted to

$$\langle\eta\rangle \sim \begin{pmatrix} \varepsilon \\ 0 \\ v_\eta \end{pmatrix}. \quad (21)$$

We do not include the shift in the VEV of φ , because $v_\eta \ll v_\varphi$, and the modification is negligible. Note that the Yukawa lagrangian that we discussed in (18) in terms of the new VEVs can be written as

$$\mathcal{L}_Y = a\bar{c}_L v_\eta u_R - a\bar{u}_L v_\eta c_R + b\bar{t}_L v_\varphi t_R - a\bar{c}_L \varepsilon t_R + a\bar{t}_L \varepsilon c_R. \quad (22)$$

Therefore, in the (u, c, t) basis, the structure of the mass matrix now is

$$\bar{m}^{\frac{2}{3}} = \begin{pmatrix} 0 & -av_\eta & 0 \\ av_\eta & 0 & a\varepsilon \\ 0 & -a\varepsilon & bv_\varphi \end{pmatrix}. \quad (23)$$

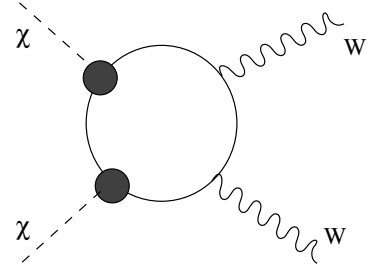


Fig. 7. Diagram leading to the coupling between two composite scalar bosons and two gauge bosons

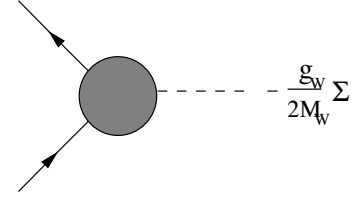


Fig. 8. Vertex coupling a scalar composite boson to ordinary fermions

This example was motivated by a system of fundamental Higgs bosons [21]. But the most remarkable fact is that we can reproduce this result with a composite system formed by the effective low energy theories coming from QCD and TC as we shall see in the following. The coefficient ε in (21) will result from the minimization of the full potential

$$V(\eta, \varphi) = \mu_\eta^2 \eta^\dagger \eta + \lambda_\eta (\eta^\dagger \eta)^2 + \mu_\varphi^2 \varphi^\dagger \varphi + \lambda_\varphi (\varphi^\dagger \varphi)^2 + \Pi \eta^\dagger \eta \varphi^\dagger \varphi + \delta \eta^\dagger \varphi \eta \varphi^\dagger. \quad (24)$$

This coefficient can be calculated if we assume that $\langle\eta\rangle$ is given by (21), $\langle\varphi\rangle$ is the same VEV as described by (16), and both are related to the TC and QCD condensates. We will neglect δ compared to Π in (20), which is reasonable if we look at Fig. 6 (Π is given by the first diagram). The coupling Π is computed from the first diagram of Fig. 6 using the effective vertex $\chi\chi WW$ shown in Fig. 7, where an ordinary fermion runs in the loop, where the χ field may indicate technicolor ($\chi = \eta$) or QCD ($\chi = \varphi$) composite scalar bosons. To compute Fig. 7 we also need the effective coupling between the composite scalar boson and the ordinary fermions. This one has been calculated in the work of Carpenter et al. [8, 9] some years ago and it is shown in Fig. 8. After a series of steps the calculation of the diagram of Fig. 7 will be given by

$$\Pi_{\chi\chi WW} \sim -\frac{g_W^4 \delta^{ab}}{M_W^2} \frac{g^{\mu\nu}}{32\pi^2} \int dq^2 \frac{\Sigma_\chi^2}{q^2}. \quad (25)$$

Following closely the procedure adopted by Carpenter et al. [9] we may approximate the self-energy by $\Sigma_\chi \sim \mu_\chi \left(\frac{q^2}{\mu_\chi^2}\right)^{-\varsigma}$, where $\varsigma_\chi = \frac{3C_{2\chi}g_\chi^2}{16\pi^2}$, to obtain the following coupling between two composite scalars and the intermediate gauge bosons of the weak interaction

$$\Pi_{\chi\chi WW} \sim -\frac{M_W^2 \delta^{ab}}{2\pi^2} \frac{G_F^2 \mu_\chi^2}{\varsigma_\chi} g^{\mu\nu}. \quad (26)$$

In (26) we made use of the relation $\frac{G_F}{\sqrt{2}} = \frac{g_W^2}{8M_W^2}$. Note that the coupling between scalars and gauge bosons is dominated by the ultraviolet limit, where the approximation for the self-energy discussed above is also valid. The effective coupling Π in (24) is equivalent to the calculation of the first diagram of Fig. 6. Using (26) we will come to the following expression:

$$\Pi_{\eta\varphi\varphi} = \frac{M_W^4 G_F^4 \mu_{TC}^2 \mu_{QCD}^2}{32\pi^8 c_{TC} c_{QCD}}. \quad (27)$$

We can now approximately determine the value of ε assuming that the potential of (14) has a minimum described by the VEVs $\langle\varphi\rangle$, (16), and $\langle\eta\rangle$, (21), which leads us to the following value of the potential at the minimum:

$$V(\eta, \varphi)|_{\min} = \mu_\eta^2 v_\eta^2 + \lambda_\eta v_\eta^4 + \mu_\varphi^2 v_\varphi^2 + \lambda_\varphi v_\varphi^4 + \lambda_\eta \varepsilon^4. \quad (28)$$

We then compare the minimum of this potential with the one obtained from (24), where the term proportional to δ is neglected in comparison to the one proportional to Π . This is equivalent to saying that the second diagram of Fig. 6 is much smaller than the first diagram, and the VEVs entering in (24) are the unperturbed ones because the perturbation will enter through the Π term. Finally, assuming that the coefficient describing the coupling between four scalar bosons that are formed in the chiral symmetry breaking of QCD is given by [9] we have

$$\lambda_\eta = \frac{G_F^2 \mu_{QCD}^4 c_{QCD} \alpha_{QCD}}{\pi}, \quad (29)$$

and we can obtain a similar expression for λ_φ after exchanging the indices QCD and TC. Equalizing v_η and v_φ to the known QCD and TC condensates (assuming $\langle\psi_i\psi_i\rangle = v^3 \approx \mu_i^3$ [22]), we conclude that

$$\varepsilon \sim B \sim \left(\frac{M_W^4 G_F^2 \mu_{TC}^4}{18\pi^3 c_{TC} \alpha_{TC}} \right)^{\frac{1}{4}} \text{ GeV} \sim 16.8 \text{ GeV}. \quad (30)$$

The surprising fact in this calculation is that the coupling of the different scalar bosons has been determined dynamically and gives exactly the expected value for the non-diagonal coefficient B .

In models with fundamental scalar bosons this value results from one ad hoc choice. In this section we presented our model, determined the main diagrams contributing to the fermion masses and showed that this scenario naturally leads to a fermion mass matrix with the Fritzsch texture. We have not tested many other models, but it seems that we may have a full class of models along the line that we are proposing here. Because of the peculiar dynamics that we are assuming we need only a horizontal symmetry and a partial unification of the standard model and the value of their mass scales will not strongly modify our predictions (although the chosen horizontal symmetry will). Of course, the breaking of the unified and/or horizontal symmetry will happen at a very high energy scale and will not be discussed here. In particular, this symmetry breaking can be even promoted by fundamental scalars which naturally can appear near the Planck scale.

4 Computing the mass matrix

We can now compute the mass matrix. Let us first consider only the $\frac{2}{3}$ charged quarks and verify their different contributions to the matrix in (1). These will come from the diagrams labeled (a), (b) and (c) in Fig. 3 and are equal to

$$\begin{aligned} A &= \frac{\mu_{QCD}}{10c_{QCD}\alpha_{QCD}} \left[1 + bg_{QCD}^2 \ln \frac{M_5^2}{\mu_{QCD}^2} \right]^{-\gamma_{QCD}+1} \\ &+ \frac{4\mu_{QCD}}{135c_{QCD}\alpha_{QCD}} \left[1 + bg_{QCD}^2 \ln \frac{M_h^2}{\mu_{QCD}^2} \right]^{-\gamma_{QCD}+1}, \\ B &= \frac{28\mu_{TC}}{675\pi c_{TC}\alpha_{TC}} \left[1 + bg_{TC}^2 \ln \frac{M_9^2}{\mu_{TC}^2} \right]^{-\gamma_{TC}+1}, \\ C &= \frac{2\mu_{TC}}{15c_{TC}\alpha_{TC}} \left[1 + bg_{TC}^2 \ln \frac{M_9^2}{\mu_{TC}^2} \right]^{-\gamma_{TC}+1}. \end{aligned} \quad (31)$$

Here the contributions for A , B and C come respectively from the diagrams (a), (b) and (c) displayed in Fig. 3. The values A , B and C correspond to the non-diagonal masses in the horizontal symmetry basis. To come to these values we assumed α_k ($=\alpha_9 = \alpha_h = \alpha_5$) $\sim \frac{1}{45}$ at the unification scale. We also assumed, when computing diagrams involving the technileptons and techniquarks condensates, the following relation:

$$\langle\bar{L}L\rangle = \frac{1}{3}\langle\bar{Q}Q\rangle, \quad (32)$$

because the techniquarks carry also the three color degrees of freedom. As the mass matrix is the same as obtained in [1] we can use the same diagonalization procedure to obtain the t , c and u quark masses, which is given by

$$M_{f\text{Diag}}^{\frac{2}{3}} = R^{-1} M_f^{\frac{2}{3}} R, \quad (33)$$

where R is the rotation matrix described in [1]. After diagonalization we obtain

$$m_u \sim \frac{|A|^2}{|B|^2} |C|, \quad m_c \sim \frac{|B|^2}{|C|} \quad \text{and} \quad m_t \sim |C|, \quad (34)$$

where the values of A , B and C are the ones shown in (31). We will also assume the unification mass scale as $M_9 = M_5 \sim 10^{16}$ GeV and the horizontal mass scale equal to $M_h \sim 10^{13}$ GeV. The several constants contained in (31) are $b_{TC} = \frac{1}{16\pi^2} \frac{26}{3}$, $b_{QCD} = \frac{7}{16\pi^2}$, $\gamma_{TC} = \frac{15}{23}$ and $\gamma_{QCD} = \frac{4}{7}$. We remember again that we assumed $\alpha_k \sim \frac{1}{45}$, $\mu_{TC} = 250$ GeV and $\mu_{QCD} = 250$ MeV. The fermion masses turn out to be functions of the parameter $c_{(TC, QCD)}\alpha_{(TC, QCD)}$. For simplicity (as well as because this is a reasonable choice) we will define $c\alpha = c_{TC}\alpha_{TC} = c_{QCD}\alpha_{QCD} = 0.5$.

We display in Table 1 the fermionic mass spectrum obtained in this model. Some of the values show a larger disagreement in comparison to the experimental values,

Table 1. Approximate values for quarks and leptons masses according to the chosen values of couplings and strongly interacting mass scales

m_t	160.3 GeV	m_b	113 GeV	m_τ	131.2 GeV
m_c	1.57 GeV	m_s	1.10 GeV	m_μ	1.30 GeV
m_u	29.6 MeV	m_d	15.6 MeV	m_e	5.5 MeV

and others show a quite reasonable agreement if we consider all the approximations that we have performed and the fact that we have a totally dynamical scheme.

It is also impressive that B in (31), neglecting logarithmic terms, is roughly given by $B \sim 14\alpha_h m_t/\pi$ which is of the order of 17 GeV. This is the expected value according to the estimate of the previous section (see (30)). In some way this is also expected in a mechanism where one fermionic generation obtains a mass at one loop level coupling to the next higher generation fermion (see, for instance, [23]). The values of the u and e masses can easily be lowered with a smaller value of μ_{QCD} . Of course, we are also assuming a very particular form for the mass matrix based in one particular family symmetry. Better knowledge of the symmetry behind the mass matrix and a better understanding of the strong interaction group alignment will certainly improve the comparison between data and theory. The high value for the masses obtained for some of the second generation fermions also come about from the overestimation of the b and τ masses. The mass splitting between the t and b quarks, which is far from the desirable result, is a problem that has not been satisfactorily solved in most of the dynamical models of mass generation up to now. It is possible that an extra symmetry, preventing these fermions to obtain masses at the leading order as suggested by Raby [24] can easily be implemented in this model. We will discuss these points again in the conclusions. Finally, considering that we do not have any flavor changing neutral current problems [26] because the interaction between fermion and technifermions has been pushed to very high energies, and that we assume only the existence of quite expected symmetries (a gauge group containing TC and the standard model and a horizontal symmetry) the model does quite well in comparison with many other models.

5 Pseudo-Goldstone boson masses

Another problem in technicolor models is the proliferation of pseudo-Goldstone bosons [2, 3, 25]. After the chiral symmetry breaking of the strongly interacting sector a large number of Goldstone bosons are formed, and only few of these degrees of freedom are absorbed by the weak interaction gauge bosons. The others may acquire small masses resulting in light pseudo-Goldstone bosons, that have not been observed experimentally. In our model these bosons obtain masses that are large enough to have escaped detection at the present accelerator energies but will show up at the next generation of accelerators (for instance, LHC). We can list the possible pseudo-Goldstone bosons according to their different quantum numbers:

Colored pseudos. These carry color degrees of freedom and can be divided into the **3** or **8** color representations. We can indicate them by

$$\Pi^a \sim \bar{Q}\gamma_5\lambda^a Q.$$

Charged pseudos. These carry electric charge, and we can take as one example the following current:

$$\Pi^+ \sim \bar{L}\gamma_5 Q,$$

where $Q(L)$ indicate the techniquark (technilepton) fields. *Neutral pseudos.* These do not carry color or charge, and one example is

$$\Pi^0 \sim \bar{N}\gamma_5 N.$$

Following closely [25], the standard procedure to determine the $SU(3)_{\text{QCD}}$ contribution to the mass (M_c) of a colored pseudo-Goldstone boson gives

$$\begin{aligned} M_c &\sim \left(\frac{C_2(R)\alpha_c(\mu)}{\alpha_{el}} \right)^{\frac{1}{2}} \frac{F_\Pi}{f_\pi} 35.5 \text{ MeV} \\ &\sim 170\sqrt{C_2(R)} \text{ GeV} \sim O(300) \text{ GeV}. \end{aligned} \quad (35)$$

While the electromagnetic contribution to the mass of the charged pseudo-Goldstone bosons is estimated to be [25]

$$M_{\text{em}} \sim Q_{\text{ps}} \frac{F_\Pi}{f_\pi} 35.5 \text{ MeV} \sim Q_{\text{ps}} 47 \text{ GeV} \sim O(50 \text{ GeV}), \quad (36)$$

in the equations above we assumed that the technipion and pion decay constants are given by $F_\Pi \approx 125 \text{ GeV}$ and $f_\pi \approx 95 \text{ MeV}$, Q_{ps} is the electric charge of the pseudo-Goldstone boson, and $C_2(R)$ is the quadratic Casimir operator in the representation R of the pseudo-Goldstone boson under the TC group. There is not much to change in these standard calculations, except that due to the particular form of the technifermion self-energy the technifermion will acquire large current masses and subsequently the pseudos-Goldstone bosons formed with these ones.

We know that any chiral current Π^f can be written as a vacuum term $m_f \langle \bar{\psi}_f \psi_f \rangle$ plus electroweak (color, ...) corrections [27], where m_f is the current mass of the fermion ψ_f participating in the composition of the current Π^f , and neglecting the electroweak corrections and using PCAC in the case of QCD we obtain the Dashen relation,

$$m_\pi^2 \approx \frac{m_q \langle \bar{q}q \rangle}{f_\pi^2}, \quad (37)$$

where $\langle \bar{q}q \rangle$ is the quark condensate. Of course this relation is valid for any chiral current, and in particular for the technifermions we can write

$$M_\Pi^2 \approx \frac{M_{T_f} \langle \bar{T}_f T_f \rangle}{F_\Pi^2}, \quad (38)$$

where M_{T_f} is the technifermion current mass. In the usual models (with the self-energy given by (3)) the technifermions are massless or acquire very tiny masses, leading to negligible values for M_Π . In our model this is not

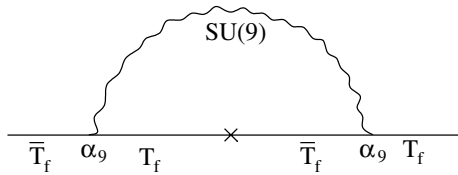


Fig. 9. Diagram responsible for the technifermion mass generation

true. All technifermions acquire masses due to the self-interaction with their own condensates through the interchange of $SU(9)$ bosons.

There are several bosons in the $SU(9)$ (and also in the $SU(3)_H$) theory connecting to technifermions and generating a current mass as is shown in Fig. 9.

A simple estimate, based on (4), of the contribution of Fig. 9 to the technifermion masses gives

$$M_{T_{f_{SU(9)}}} \gtrsim O(80-130) \text{ GeV}. \quad (39)$$

If we also include the contribution of the same diagram where the exchanged boson is a horizontal $SU(3)_H$ boson coupling technifermions of different generations, we must add to the above value the following one:

$$M_{T_{f_{SU(3)_H}}} \gtrsim O(10-40) \text{ GeV}. \quad (40)$$

Therefore, we expect that the technifermion current masses are at least of the order of $M_{T_f} \approx O(100) \text{ GeV}$. Now, according to (38) and assuming $\langle \bar{T}_f T_f \rangle \sim F_H^3$ we have the following estimate for the pseudo-Goldstone boson masses:

$$M_H \gtrsim O(100) \text{ GeV}. \quad (41)$$

Note that in this calculation we have not considered the QCD or electroweak corrections discussed previously. Therefore, even if the pseudo-Goldstones bosons do not acquire masses due to QCD or electroweak corrections they will at least have masses of order of 100 GeV because of the “current” technifermion masses obtained at the $SU(9)$ (or $SU(3)_H$) level.

6 Conclusions

We have presented a technicolor theory based on the group structure $SU(9) \times SU(3)_H$. The model is based on a particular ansatz for the TC and QCD self-energy. We argue that our ansatz for QCD, in view of the many recent results about its infrared behavior, is a plausible one, but even if it is considered as an “ad hoc” choice for the self-energy the main point is that it leads to a consistent model for the fermion masses. This is the only new ingredient in the model; all the others (unification of TC and the standard model and the existence of a horizontal symmetry) are naturally expected in the current scenario of particle physics. One of the characteristics of the model is that the first fermionic generation basically obtain masses due to the interaction with the QCD condensate, whereas the third

generation obtains masses due to its coupling with the TC condensate. The reason for this particular coupling and for the alignment of the strong theory sectors generating intermediate masses is provided by the $SU(3)_H$ horizontal symmetry. Of course, our model is not successful in predicting all the fermion masses, although it has a series of advantages. It does not need the presence of many ETC boson masses to generate the different fermionic mass scales. The ETC theory is replaced by unified and horizontal symmetries. It has no flavor changing neutral currents or unwanted light pseudo-Goldstone bosons. There are many points that still need some work in this line of the model. The breaking of the $SU(9)$ and horizontal symmetries is not discussed and is just assumed to occur near the Planck scale and possibly could be promoted by fundamental scalar bosons.

The mass splitting in the third generation could be produced with the introduction of a new symmetry. For instance, if in the $SU(9)$ breaking besides the standard-model interactions and the TC one we leave an extra $U(1)$, maybe we could have quantum numbers such that only the top quark would be allowed to couple to the TC condensate at leading order. This possibility should be further studied because it also may introduce large quantum corrections in the model. If the unified group ($SU(9)$ in our case) is not broken by a dynamical mechanism, i.e. we do not need that this group tumbles down to $SU(4)_{TC} \otimes \text{SM}$, then we could replace $SU(4)_{TC}$ by one smaller group (perhaps $SU(2)_{TC}$) which becomes stronger at the scale $\mu \approx 250 \text{ GeV}$.

In this class of models we can choose different groups containing TC and the standard model as well as different horizontal symmetries with different textures for the mass matrix. These will certainly modify the values of the fermion masses that we have obtained. The alignment of the strongly interacting sectors can be studied only with many approximations, but it is quite possible that it generates more entries to the mass matrix than only the term B . Another great advantage of the model is that it is quite independent of the very high energy interactions (like $SU(9)$ or $SU(3)_H$), although the horizontal symmetry is fundamental to obtain the desired mass matrices, and we believe that variations of this model can be formulated.

Acknowledgements. This research was supported by the Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq) (AAN) and by Fundação de Amparo à Pesquisa do Estado de São Paulo (FAPESP) (AD).

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